

# The Care and Feeding of a Physics Problem Set

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# 1 Introduction

Dear Student:

The document you hold in your hands is quite important. It contains detailed instructions on how to raise your problem set so that it grows up to be strong, healthy, and high scoring. As an added benefit, the suggestions in this document may even aid the learning process. In the sections that follow I will describe various components which make for a good solution. These rules are not arbitrary nor meant solely for my benefit. They are things which I started doing over the course of my college career, sometimes voluntarily and other times involuntarily, and which helped me master the material. I will therefore explain my rationale for including them. I hope you will approach them with an open mind and try to see how they are helpful.

The above table of contents doubles as a summary of the components of a good problem set. As the name suggests, the required components in Section 2 are expected, and you will lose points if you fail to incorporate those elements into your homework. The recommended components in Section 3 will, in my judgment, greatly benefit you, but I will not mark off for their absence. The very short Section 4 lists components which carry extra credit (at present it has one element, 4.1). In order to clarify the rules, Section 5 gives a sample problem with different solutions. It then explains how each of these solutions might be graded. Finally, some last thoughts appear in Section 6.

## 2 Required Components

### 2.1 Explain your work clearly, in English

*English is the water for your problem set: it is absolutely essential for survival. Like physical water, it is possible to drink too much, but no one ever does.*

This is the single most important part of a good solution. You should assume that I do not know how to solve the problem and that you need to explain it to me. If your solution differs significantly from mine, this assumption may actually be correct. You will not receive full credit if I do not completely understand how you arrived at your answer. This is because getting the correct answer does not indicate that your method of solution is correct, as shown by the following classic calculation:

$$\frac{16}{64} = \frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}.$$

It is an unfortunate fact that there are many wrong ways to get the right answer. Explaining your work so that I understand your logic is the only way to convince me that you got the right answer by valid means.

However, explaining your work clearly has benefits for you completely unrelated to your grade. As you write up your solution you may find that you do not fully understand certain steps. By writing as precise a solution as possible, you will discover exactly what parts of the material still confuse you, and will thus be in a better position to seek help (from the book, a friend, or myself). You may also find that steps which seem obvious to you when you were solving the problem are

no longer obvious when you are looking at your work later, for example when you are studying for the test. By writing everything down you will help yourself understand what you are doing.

## 2.2 Have a high E/M (English/Math) ratio

*Just as you should drink at least 2 quarts of water each day, your problem set needs an E/M ratio of at least 2 for optimal health.*

This is really the same as 2.1, but it is sufficiently important that it is worth saying a second time. A substantial portion of your solution should be in English. A sequence of equations by itself transmits little information. Short helper phrases like “therefore,” “on the other hand,” “by definition,” and “substituting in from... yields” can work wonders in clarifying the logical structure of a solution. My solutions almost never have an E/M ratio less than one, and it typically hovers between 2 and 3 (in terms of vertical space consumed by each on the page). Your solutions should be comparable.

## 2.3 Indicate your assumptions explicitly

*Problem sets should always be vaccinated against unclear wording, hidden assumptions, and rabies.*

It is a sad truth that physics problems are not always as precisely stated as they should be. Authors may make hidden assumptions which are necessary to make the problem tractable. Other times, they use various code phrases to indicate precise assumptions or conditions. For example, “small...” typically means “keeping only the first term in the Taylor series in...”. By stating your assumptions up front, you help me understand your solution, especially if you interpreted the problem differently than I did. More importantly, you may be cluing yourself in to the fact that you can use simplification techniques such as the binomial expansion.

## 2.4 Define your variables

*Always start off your problem set's day with a balanced breakfast of variable definitions.*

Define your variables unless they are defined in the problem statement. Almost every variable has multiple “conventional” uses. Even if it does not, different people will usually manage to find different conventions for a given variable, such as different origins, different orientations of the axes, or even different parts of the system. You may even find that you no longer remember which conventions you used when you reread your solution later. Therefore, when in doubt, define, as this increases the odds that I will understand your solution correctly. Including appropriate illustrations of your variable definitions is extremely helpful in this regard.

## 2.5 Solve the problem symbolically

*Even though your problem set may want to run ahead to the numbers in the park, it needs to learn to stay with you as you walk along the symbolic trail.*

While there are occasional exceptions, numbers have no business appearing until the last line of a solution. Even in exceptional cases, a variable should be assigned to the intermediate answer rather than simply using the number. After all, if the numbers are plugged in, there is no way to take limits as described in 3.4, below. Non-symbolic computations also make it more

difficult to follow the logic and reduce the opportunity for partial credit. If you just have a horribly wrong number without any symbolic expressions, I may have no idea whether you simply punched the numbers incorrectly into your calculator or whether your entire method of solution is incorrect.

## 2.6 Use a reasonable number of digits

*Digits form the fat of a problem set's diet: some is absolutely essential, but too much will quickly lead to poor health.*

The final answer should have the number of significant digits dictated by the quantities in the problem statement. This is rarely more than three, no matter how many digits the calculator gives. Never write a number with twelve digits. The answer is not 0.99999999999978; rather, it is 1.00 (or possibly  $2.2 \times 10^{-13}$  if the requested result is the deviation from unity). This both expresses the correct number of significant digits and is much easier to read. Besides clarity, keeping track of significant digits will often indicate at what point a particular expansion, such as the binomial expansion, can be truncated. By this I mean that if the answer has three significant digits, and the second order correction is of order  $10^{-6}$ , you can be certain that you may safely truncate the series at first order.

## 2.7 Apply the “Gravity Does Not Point Up” test

*Pay attention to the behaviour of your problem set. If it becomes strange or unexpected, it may be ill and require medical attention.*

A professor once told me, “gravity does not point up. If you finish a problem and discover that gravity is pointing up, you’ve probably done something wrong.” The wonderful thing about physics is that it deals with the world around us, and thus we often have clues as to what the answer should be. After you have gotten your answer, pause and consider its reasonableness. If you computed that the lifetime of an atomic transition is  $10^{10}$  times the age of the universe, you have likely made an error. Go back and look for that error. If you cannot find it, indicate that your answer is unreasonable and why. Such a statement indicates a greater level of understanding of the material, and hence receives more partial credit, than blindly writing down a clearly wrong answer.

## 2.8 Get your units right

*If your problem set exhibits the aggression of conflicting units, seek the help of your veterinarian immediately!*

The first step of the “Gravity” test is to make sure the units of your answer are right. If the problem asks for a velocity, the units cm/s, m/s, and furlongs/fortnight are all possibly correct, but kg-m/s is not. Answers which have the wrong dimensions are necessarily wrong—go back and check what happened. Further, numbers should always have their units attached to them (unless they are dimensionless). This makes it much easier to find out where the units went bad. Notice that you do not need to plug in numbers to see if the units in your answer are right. Each variable has a dimension associated to it. Dimensional analysis of your symbolic answer should tell you if it has the right units. If it does not, check the dimensional consistency of each previous equation.

## 3 Recommended Components

### 3.1 Copy the problem statement

*Problem sets are happiest when all their parts can be found in one place*

Put a copy of the problem on the same page as your solution. This can be either a literal copy which is pasted (or transcribed) onto the paper or a paraphrasing of the problem statement. The advantage of the former is that it is quicker and ensures that the copy is correct. The advantage of the latter is that the process of paraphrasing may help you understand the problem better. In either event, attaching a copy makes studying for exams easier, as both problem and solution can be found in one place.

### 3.2 Use only one side of the paper

*Problem sets enjoy pet doors which allow them to move freely from the house to the back yard.*

Using one side of the paper makes it easier to follow a solution if it stretches across more than one page. However, if you worry about killing trees, do not hesitate to use both sides of the paper.

### 3.3 Include proper diagrams whenever possible

*Supplement your problem set's diet with diagrammatic multivitamins when necessary. Small but powerful, a picture can be worth a thousand milligrams of words.*

A proper diagram (i.e., one that is legible, has labeled axes, and is drawn with sufficient care to convey the essential quantities in the problem) can immediately clarify your variable definitions for me and provide insight into the problem for you. Sometimes there is a temptation to solve problems entirely algebraically, but a diagram can instantly reveal the relationship between variables and make answers obvious. When in doubt, draw.

### 3.4 Examine various limits of your answer

*Kant's categorical imperative applies just as well to pet care as it does to moral philosophy: things which are good for your pet in one situation should still be good for it in many other situations.*

One of the best ways to test the reasonableness of your answer is to take various limits of the final answer. It may not always be possible to look at the answer and determine whether it is sensible. However, there will usually be special cases in which the answer should be easy to understand. Figure out what happens to your answer when one of the masses goes to zero, when the two masses become equal, or when the incline plane becomes vertical. Taking these limits is a basic sanity check that practicing physicists use every day. Further, it often provides intuition into “what’s going on” as you see which pieces of your answer contribute in the different regimes.

### 3.5 Explain what you *would* do

*Sometimes your problem set will be unhappy no matter what you do to cheer it up; while you need to accept these situations, you should not allow your problem set to just wallow in its sorrow.*

If you get stuck on a particular part of a problem, do not simply give up. Go as far as you can with that part, then explain why you are stuck (for example, “I don’t know how to evaluate this integral”). Continue with the rest of the problem and describe what you would do if you could get past the obstacle. I do give partial credit for such descriptions. Further, the process of writing down why you are stuck may clarify the problem and help you solve it.

## 4 Bonus Components

### 4.1 $\text{\LaTeX}$ your homework

*While problem sets will often balk at being typeset, three or four treats will usually convince them to cooperate.*

As you advance in your physics education and your eventual career, you will be expected to produce reports, long papers, and other technical documents. It will be to your advantage to know how to produce them.  $\text{\LaTeX}$  is the standard program for producing papers in the physics community and its use is highly recommended. However, any computer typesetter is acceptable. Because this requires significantly more time than simply hand-writing solutions, computer-generated homework carries an automatic 10% bonus. **Note:** MS Word is **not** a typesetter.

## 5 Examples

In this section I present three different solutions to a problem. After each problem comes a discussion of what grade that solution would receive and why. These examples should make very clear what sort of solutions I expect.

### 5.1 Problem statement

One of the most popular moves performed by ice skaters is the Axel, in which the skater pushes off the front leg and spins in the air. The Axel comes in three varieties: the single, double, and triple, corresponding to 1.5, 2.5, and 3.5 complete rotations in mid air. Skaters always extend their arms in preparation for the Axel, then bring them in while in the air. It has been observed that skaters bring their arms in sooner when performing the double or triple Axel, as compared with the single Axel. Explain why this is the case.

### 5.2 Solution 1

If we treat air resistance as negligible, then the only force acting on the skater in mid-air is gravity. Gravity acts at the center of mass, which should be very close to the axis of rotation. Thus, gravity exerts very little or no torque on the skater about the axis of rotation. This means that the angular momentum  $L$  of the skater is constant while in the air. Angular momentum for a system of particles is given by

$$L = I\omega,$$

where  $\omega$  is the angular velocity and  $I$  is the moment of inertia. The general formula for moment of inertia is

$$I = \sum_i r_i^2 m_i,$$

where  $r_i$  is the distance of the  $i^{\text{th}}$  particle from the axis of rotation and  $m_i$  is its mass. When the skater brings his or her arms in, the masses  $m_i$  stay the same, but the distances  $r_i$  become smaller. Hence the moment of inertia is decreased. Since  $L$  is conserved, this means that  $\omega$  must increase to compensate. Thus, the skater's rotation speed increases. By bringing in the arms sooner, the skater achieves this higher angular velocity earlier, which makes it easier to complete the additional revolutions for a double or triple Axel.

### 5.3 Comments on solution 1

The above solution is perfect and would score 10/10. The overall logic is clearly expressed in English (Section 2.1); the relationship between each equation is indicated in words, resulting in a high E/M ratio (Section 2.2); it states its assumptions upfront (Section 2.3); all variables, including the standard ones  $L$  and  $\omega$ , are defined as they are used (Section 2.4); and the problem is solved symbolically all the way through (Section 2.5). Since the problem asked for an explanation rather than a number, it is perhaps natural that the logic is explained in such detail, but you should strive for this level of clarity in all your solutions.

### 5.4 Solution 2

$$L_i = L_f \quad L = I\omega = (\sum_i r_i^2 m_i)\omega \quad \omega_f > \omega_i.$$

### 5.5 Comments on solution 2

Although the correct equations appear, this solution would rate only 2/10. Its greatest deficiency is that it provides no explanation of the logic (Section 2.1). Not only does it have an E/M ratio of zero (Section 2.2), it does not even bother to use an implies symbol,  $\Rightarrow$ , to show that the inequality follows from the preceding equations. Further, none of the variables are defined (Section 2.4). While the usage of  $L$ ,  $m$ , and  $\omega$  is reasonably "standard," I am forced to guess what the subscripts stand for (notice that the subscript  $i$  is used with different meanings in the two equations). Reading this solution later, most people would have a difficult time reconstructing what was done or why. This is the type of solution to avoid.

### 5.6 Solution 3

When the skater goes into the air, the only force acting on him or her is gravity, which does not exert a torque about the center of mass, so their speed of rotation is constant. When they bring their arms in, that provides an additional torque, which speeds up their rotation. When they do this earlier in the jump, they can rotate farther and complete a double or triple Axel.

### 5.7 Comments on solution 3

This solution has some deficiencies. First and foremost is a major physics error: it identifies the act of bringing in the arms as providing an additional torque rather than looking at conservation of angular momentum. Still, the overall logic of the solution is clear, and the solution does correctly identify that the purpose is to increase rotational speed. This solution would get a 4/10. Notice that this is higher than solution two, which got the "right answer" but with very poor explanation.

## 6 Final Thoughts

Solving a problem set demands a great deal of effort. The foregoing rules are intended to help you reap the maximum benefit from that effort, in terms of both your mastery of the material and your grade. Should you have any questions about any of the guidelines, or about how a particular problem was graded, you should not hesitate to come ask me.

Good luck on your problem sets!

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