The Hitchhiker's Guide to Ratio Problems

0. Don't panic! ☺

1. Identify the problem as a ratio problem. Sometimes problems ask you to *compare* two quantities, to determine *by what factor* they increase or decrease, or for the *fractional change*. These are all examples of ratio problems.

2. Give a name to each of the two quantities to be compared. Be sure to use appropriate subscripts to distinguish them.

3. Write down an equation for each of the two quantities. Again, be sure to use appropriate subscripts.

4. Find relationships relating quantities in the first system to quantities in the second. For each variable that appears in your equation, there must be an equation relating that variable in the two systems (possibly the implicit equation $q = q_A = q_B$ for a quantity q that is the same). If you cannot find relate all the quantities, you need to return to the previous step.

5. Divide the equation for the first system by the equation for the second system.

6. Substitute in the relationships found and solve for the desired ratio.

Example

Objects A and B have the same mass, but A is moving twice as fast as B. Compare their kinetic energies.

1. We are asked to compare the two kinetic energies, so we need to find a ratio. Since we are not told which ratio to solve for, we can solve for either $\mathrm{KE_A}/\mathrm{KE_B}$ or $\mathrm{KE_B}/\mathrm{KE_A}$ (both are valid comparisons). We will solve for the former.

2. We already did this: we want to compare $\mathrm{KE_A}$ and $\mathrm{KE_B}$.

3. The formula for KE is $\mathrm{KE} = \frac{1}{2}mv^2$, so we write $\mathrm{KE_A} = \frac{1}{2}m_A v_A^2$ and $\mathrm{KE_B} = \frac{1}{2}m_B v_B^2$.

4. We told that the masses are the same, so we have that $m_A = m_B$. We also know that the speed of A equals to speed of B, so $v_A = 2v_B$ (magnitude of velocity or speed).

5. We divide the left side by the left side and the right side by the right side:
$$\frac{\mathrm{KE_A}}{\mathrm{KE_B}} = \frac{\frac{1}{2}m_A v_A^2}{\frac{1}{2}m_B v_B^2} = \frac{m_A v_A^2}{m_B v_B^2}.$$
Notice that the left hand side is already the desired ratio.

6. We substitute in $m_A = m_B$ and $v_A = 2v_B$:
$$\frac{\mathrm{KE_A}}{\mathrm{KE_B}} = \frac{\cancel{m_B}\,(2v_B)^2}{\cancel{m_B}\,v_B^2} = \frac{4\,\cancel{v_B^2}}{\cancel{v_B^2}} = 4.$$
A's kinetic energy is four times as great as B's.

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