Causet Dynamics with $\frac{Dq}{Dg}$.

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17th Midwest Relativity Meeting

November 2, 2007

Motivation

Reminder

Points p and q in a (causal) spacetime will obey one of three relations: p is to the future of q, to the past of q, or spacelike related to q.

Theorem (Malament)

The points of a manifold (M, g) together with their causal structure specify the pair (M, g) up to conformal equivalence.

In 4d, this gives topology and 9 of 10 metric components.

Going Discrete

- "Missing" metric component is local volume element
- If discretizing spacetime, natural to assume one point per Plank volume
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- Given two points, only finitely many points in causal diamond.
- Causal sets naturally encode geometry with a minimal physical hypothesis.

Causets

Definition (Causal Set)

A causal set, or causet, is a point set C together with a locally finite partial order.

"Locally finite" means $\big|\{z \mid x < z < y\}\big| < \infty \; \forall\, x, y \in C$.

Classical Dynamics???

- Need to implement Einstein's equations somehow
- Eventually lead to quantum dynamics using sum over histories
- Implemented as a stochastic growth model of n-element causet to $n + k$ element causet.

Classical Sequential Growth

- Rideout and Sorkin classified all models which obey
  1. general covariance
  2. Bell causality
- Two basic varieties:
  1. transitive percolation
  2. everything else

Classical Sequential Growth

- Rideout and Sorkin classified all models which obey
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- Two basic varieties:
  1. transitive percolation ← simple, long-studied, one parameter: p
  2. everything else

Classical Sequential Growth

- Rideout and Sorkin classified all models which obey
  1. general covariance
  2. Bell causality
- Two basic varieties:
  1. transitive percolation
  2. generalized percolation ← more challenging

Generalized Percolation

Given parameters nonnegative $\{t_1, t_2, t_3, \ldots\}$

1. Choose $0 \leq k \leq n$ with relative probability $t_k \begin{pmatrix} n \\ k \end{pmatrix}$
2. Randomly choose "proto-precursor" of k elements with uniform probability.
3. Enforce implied relations using transitive closure.

Who We Are

Definition ($\frac{Dq}{Dg}$)

Discrete Quantum Dynamics Group.

- Luca Bombelli (University of Mississippi)
- Julio Tafoya (University of Mississippi)
- Itai Seggev (Knox College)
- Sam Watson (University of Mississippi)

Physics Goals

1. Perform a wide variety of analyses on a Causet
   - Myrheim Meyer dimension
   - midpoint dimension
   - ordering fraction
   - number and location of posts
   - height, width (future)
2. Compute quantities locally as well as globally.
3. Explore some parameter space of generalized percolation.
4. Possibly add other dynamics.

Programming Goals

1. Easy to add percolation "types" (functions for generating t_n).
2. Minimal external dependencies.
3. Portable.
4. Elegant.
5. Reasonably efficient, but not at the cost of the above.

Basic Choices

- Language: C++
- Use GSL for random number generator and $\Gamma(z)$.
- Implement our own "large double" type.
- Representation of causet hidden inside of class

  $\Rightarrow$ all analysis functions access the causet using class methods

- At present time, represent causet as an adjacency matrix implemented as a `vector< vector<bool> >`.

The Generalized Percolation Solution

Achtung!

This is the computer geek slide. Our regularly scheduled physics program will resume on the next slide.

1. Define a class `coupling_t` which has a "pure virtual function" called `nthconstant`

```
class coupling_t {
 public:
   virtual large_double_t nthconstant(int n)=0;
};
```

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1. Define a class `coupling_t` which has a "pure virtual function" called `nthconstant`
2. To define a new coupling type, define a class which inherits from `coupling_t`

```
class allones_t : public coupling_t {
 public:
   allones_t() {one=double_to_ldt(1.0);}
   large_double_t nthconstant(int n) {return one;}
 private:
   large_double_t one;
};
```

The Generalized Percolation Solution

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1. Define a class `coupling_t` which has a "pure virtual function" called `nthconstant`
2. To define a new coupling type, define a class which inherits from `coupling_t`
3. The general percolation method takes as an argument a `coupling_t*`, so the coupling type is determined at run time.

The Generalized Percolation Solution

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```
class coupling_t {
 public:
   virtual large_double_t nthconstant(int n)=0;
};

class allones_t : public coupling_t {
 public:
    large_double_t nthconstant(int n) {return one;}
...
};

class causet_t {
 public:
    void seed_causet_gen_perc(coupling_t *couplingConsnats);
...
};
```

A New Proof

- Alon, Bollobas, *et al.* proved that for transitive percolation and any $p > 0$, infinitely many posts occur with probability 1.
- As part of our preparation for this project, we came up with a significantly simplified "by-hand" proof.
- Forthcoming paper.

Sample Output

The program ./csg_simulation was compiled from an unmodified copy of revision 80 and was run with arguments -N 5000 --transitivePercolation --probability 0.3 --p ostsAll --myrheimMeyerDimension --midpointDimension --orderingFraction

The number of posts is 9.

The posts are 877, 3114, 3115, 4209, 4210, 4211, 4354, 4355, and 4356.

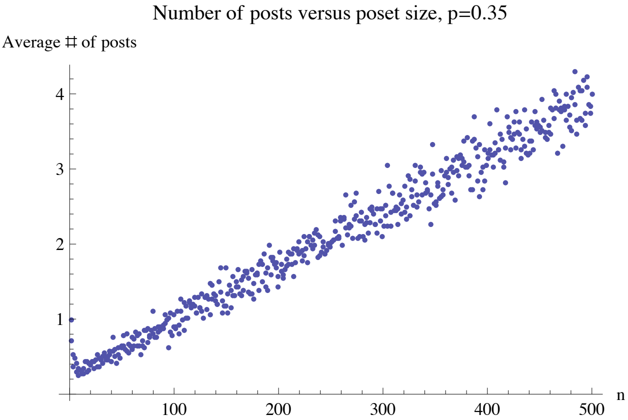
Posts			Volume	MSD	MMD	Ordering Fraction
877	-	3114	2237	1.000645	1.007679	0.995737
3114	-	3115	1	1.000000	1.000000	1.000000
3115	-	4209	1094	1.007927	1.015933	0.991146
4209	-	4210	1	1.000000	1.000000	1.000000
4210	-	4211	1	1.000000	1.000000	1.000000
4211	-	4354	143	1.040642	1.105073	0.942988
4354	-	4355	1	1.000000	1.000000	1.000000
4355	-	4356	1	1.000000	1.000000	1.000000

The midpoint scaling dimension is 1.002894.

The 2-chain abundance Myrheim-Meyer dimension is 1.003446.

The ordering fraction is 0.998086.

Sample Data


Number of posts versus poset size, p=0.35
Average ♯ of posts
1
2
3
4
100
200
300
400
500
n

What Works

- Transitive percolation (nothing new)
- Proof-of-concept generalized percolation with t_n simulating transitive percolation or $t_n = 1 \, \forall \, n$.
- Non-sequential Minkowski "sprinkling".
- Finding posts
- Dimension estimation, both locally and globally.

Current and Planned Work

1. Analyzing transitive percolation using local estimates (ongoing)
2. Bug-squashing (ongoing)
3. Analyzing more interesting general percolation models
4. Height and width calculation

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5. Suggestions and questions?