Posts in Transitive Percolation

First Results from $\frac{Dq}{Dg}$

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Who We Are

Definition ($\frac{Dq}{Dg}$)

Discrete Quantum Dynamics Group.

- Luca Bombelli (University of Mississippi)
- Itai Seggev (Knox College)
- Sam Watson (University of Mississippi)

Motivation

Reminder

Points p and q in a (causal) spacetime will obey one of three relations: p is to the future of q, to the past of q, or spacelike related to q.

Theorem (Malament)

The points of a manifold (M, g) together with their causal structure specify the pair (M, g) up to conformal equivalence.

In 4d, this gives topology and 9 of 10 metric components.

Going Discrete

- "Missing" metric component is local volume element
- If discretizing spacetime, natural to assume one point per Plank volume
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- Given two points, only finitely many points in causal diamond.
- Causal sets naturally encode geometry with a minimal physical hypothesis.

Causets

Definition (Causal Set)

A causal set, or causet, is a point set C together with a locally finite partial order.

"Locally finite" means $\left|\{z \mid x < z < y\}\right| < \infty \; \forall \, x, y \in C$.

Classical Dynamics???

- Need to implement Einstein's equations somehow
- Eventually lead to quantum dynamics using sum over histories
- Implemented as a stochastic growth model of n-element causet to $n + k$ element causet.

Classical Sequential Growth

- Rideout and Sorkin classified all models which obey
  1. general covariance
  2. Bell causality
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- Two basic varieties:
  1. transitive percolation ← simple, long-studied, one parameter: p
  2. everything else

Classical Sequential Growth

- Rideout and Sorkin classified all models which obey
  1. general covariance
  2. Bell causality
- Two basic varieties:
  1. transitive percolation
  2. generalized percolation ← more challenging

Transitive Percolation

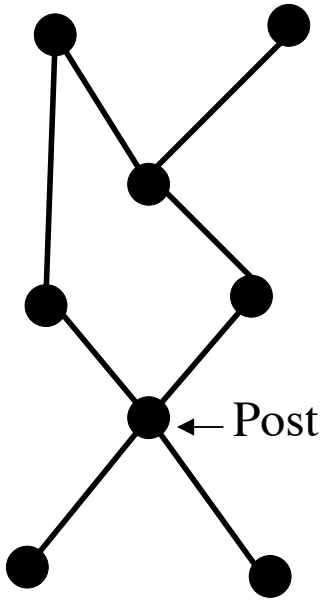
Given a probability p:

1. sprinkle n vertices;
2. for each pair (i, j) with (i, j), draw an edge from i to j with probability p; and
3. enforce implied relations using transitive closure.

Posts

Definition (Post)

A post is a point which is in the future or past of every other point.



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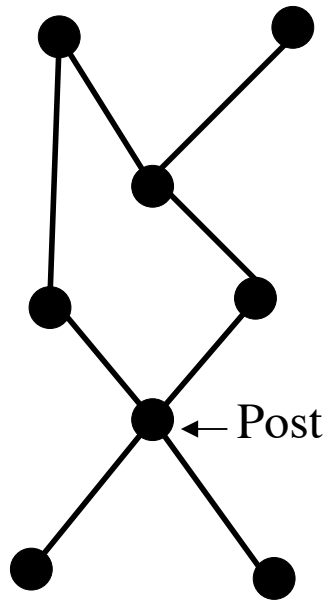
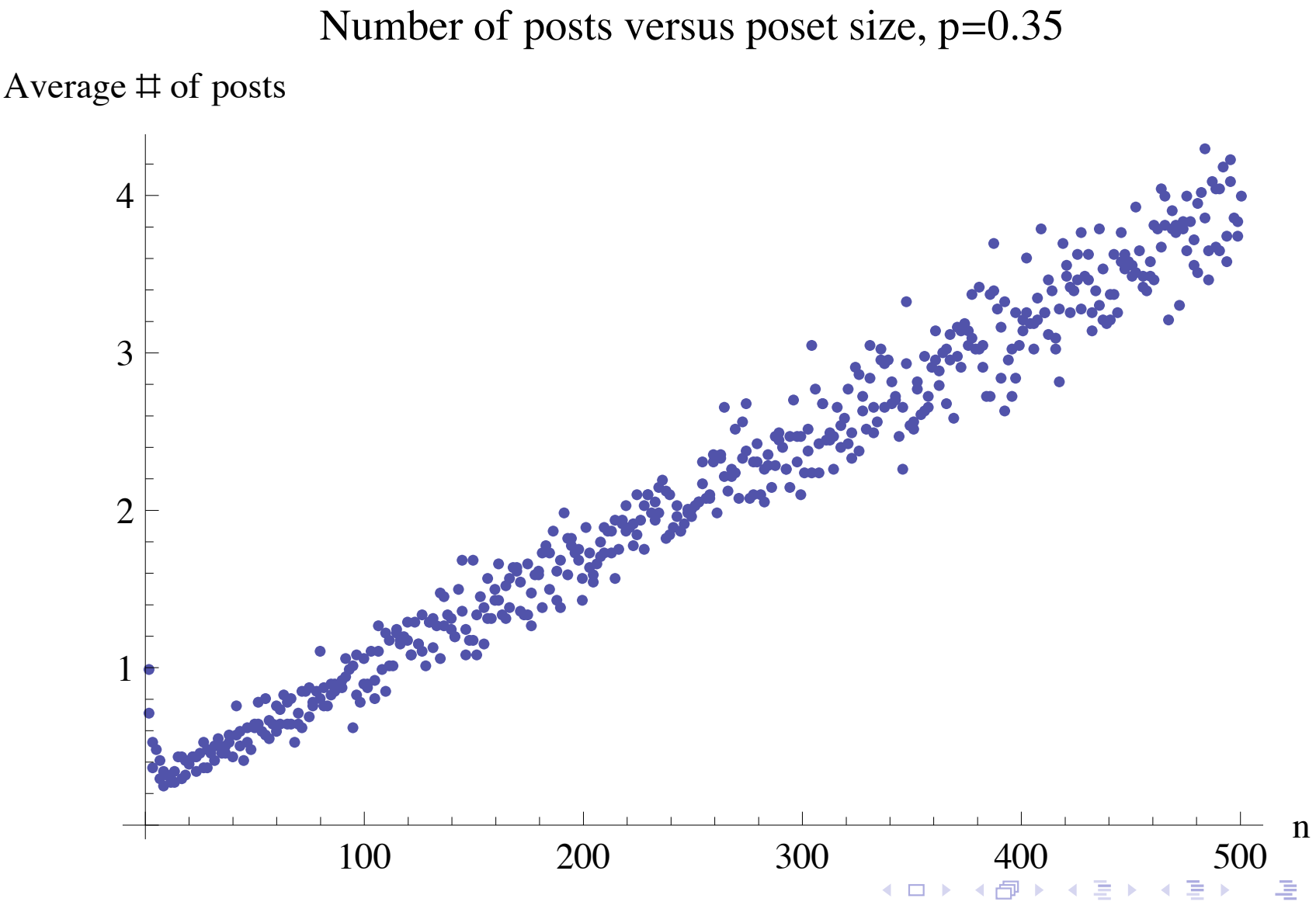
Feature, not bug!

The distance between posts gives the time between successive big crunches.



Data Last Year



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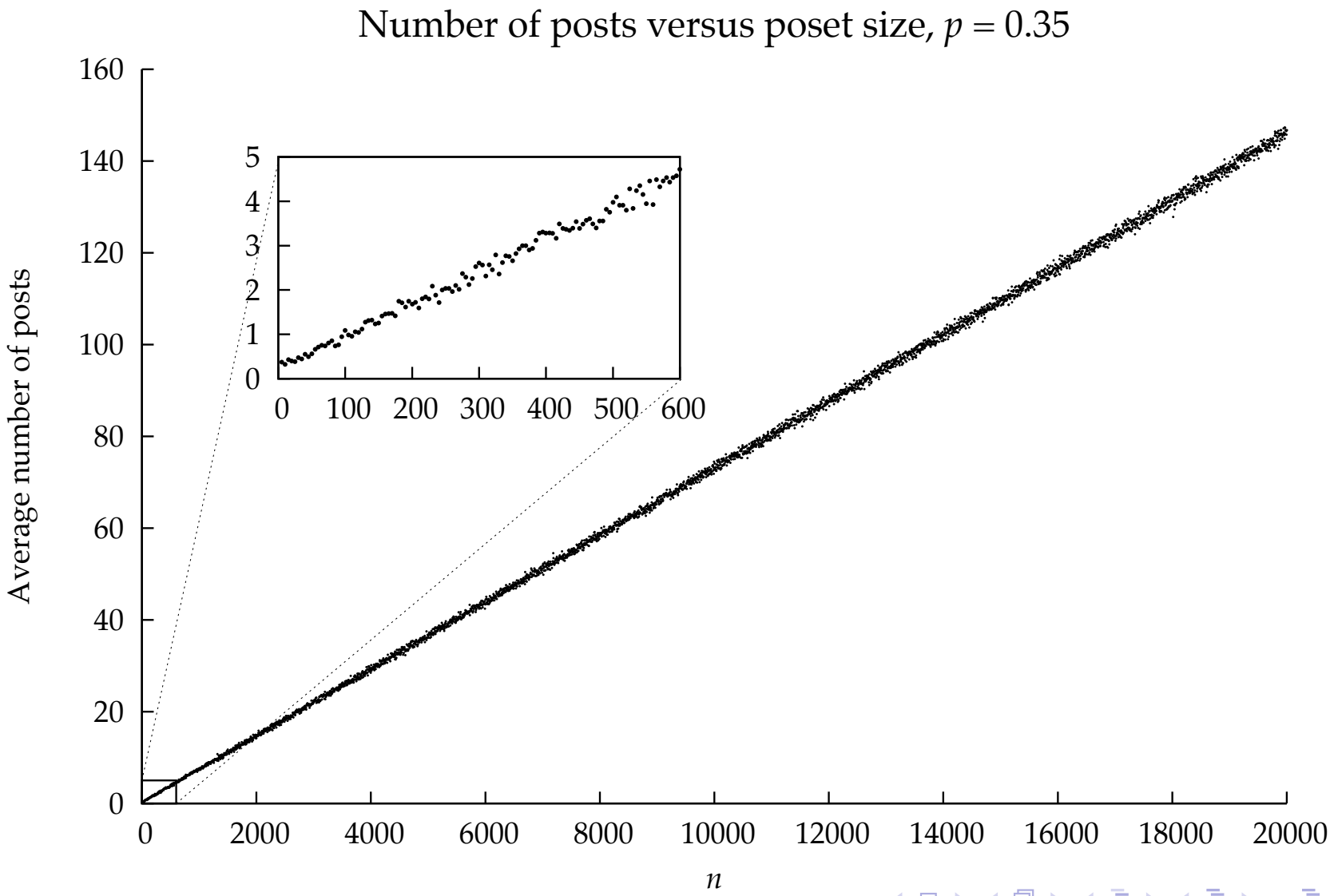
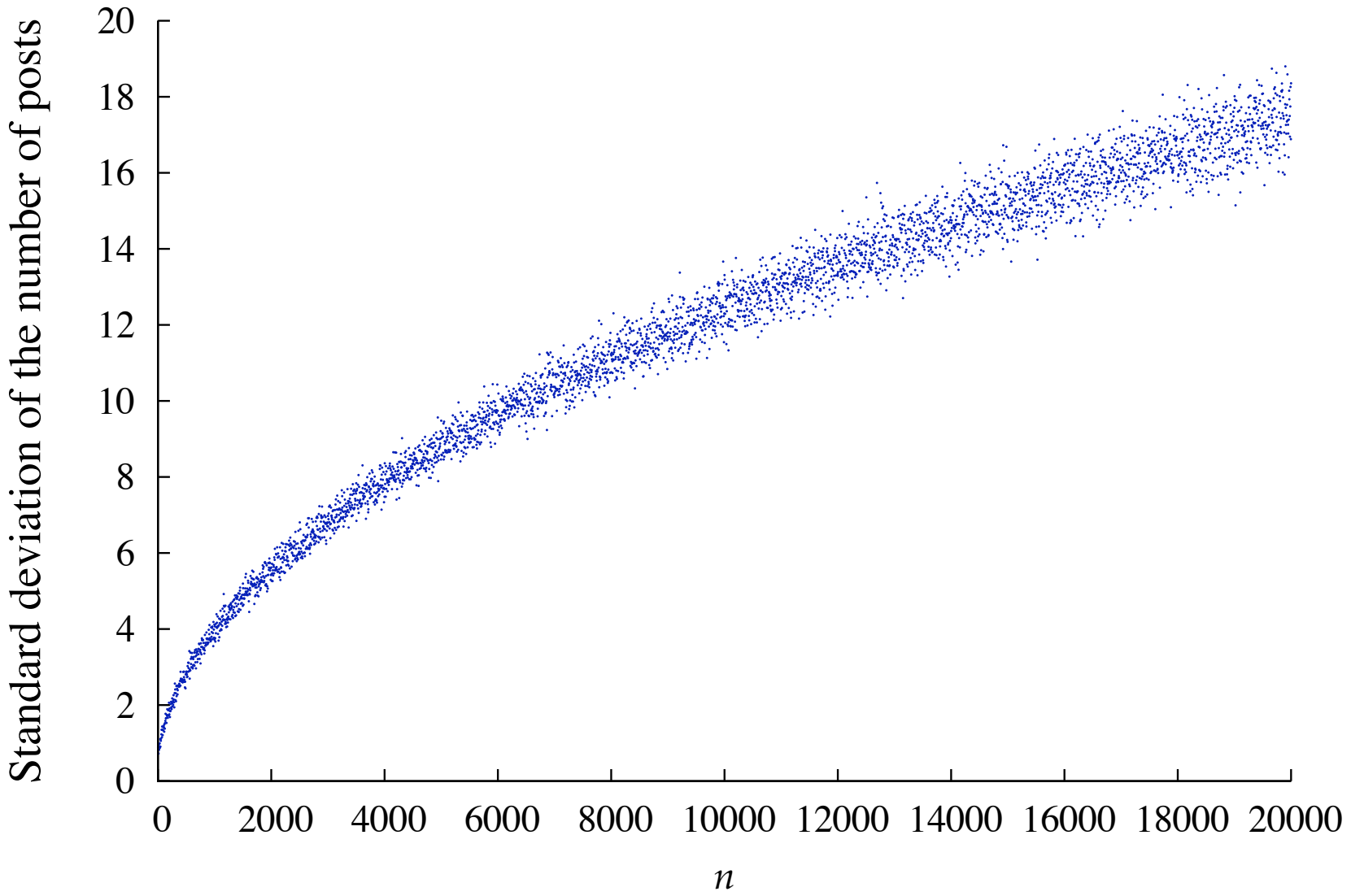
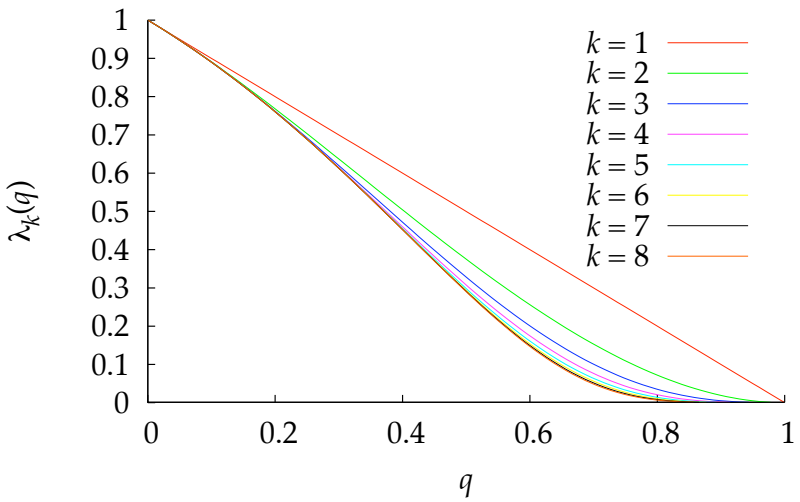


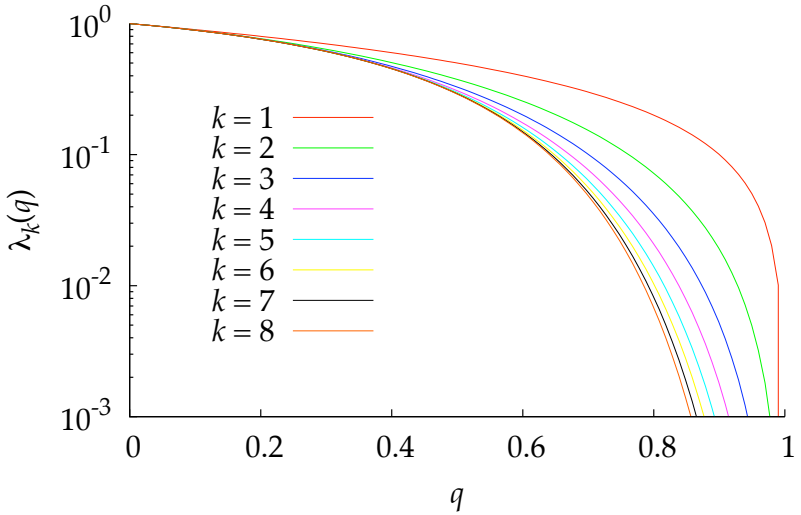
The Standard Deviation



Notation

1. $q = 1 - p$.
2. $\lambda_k(q) = \prod_{n=1}^{k}(1 - q^n)$
3. $\lambda(q) = \lim_{k \to \infty} \lambda_k(q) =$ "Euler's Function."





A Theorem

Theorem (Bombelli, Seggev, and Watson 2008)

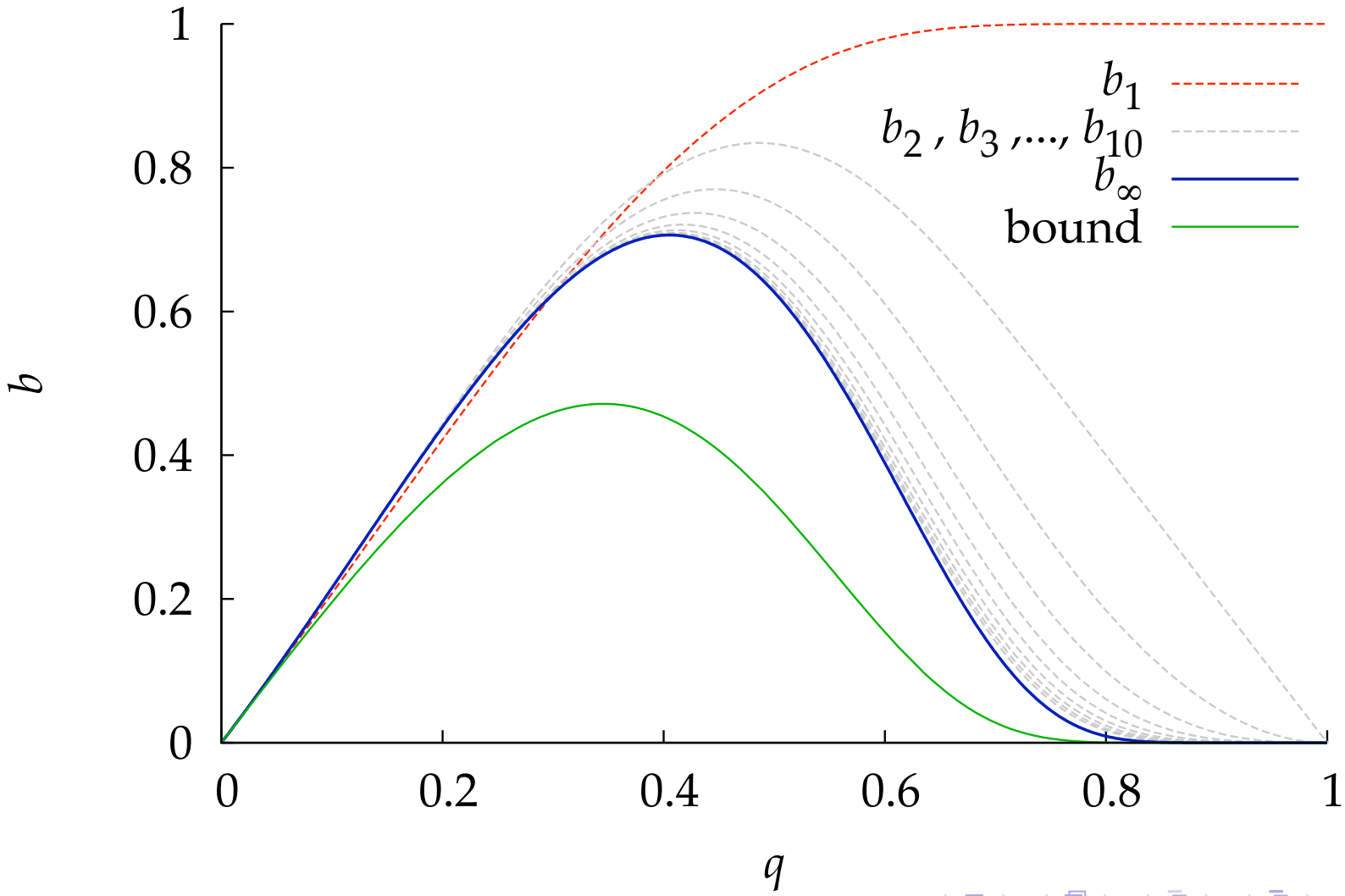
For all $0 < q < 1$, there exists a sequence of real numbers $\{b_n(q)\}_{n=1}^{\infty}$ so that for all $n \geq 1$, $b_n(q)$ is strictly between 0 and 1 and the expectation value $\langle N_{n,q} \rangle$ of the number of posts in a random graph order on $\{1, 2, 3, \ldots, n\}$ generated with probability $p = 1 - q$ satisfies

$$\langle N_{n,q} \rangle = \lambda^2(q) \cdot n + b_n(q)\,. \tag{1}$$

Moreover, $\{b_n(q)\}_{n=2}^{\infty}$ is strictly monotonically decreasing to a positive limit $b(q)$ given by the expression

$$b(q) = 2\lambda(q) \sum_{k=0}^{\infty} (\lambda_k(q) - \lambda(q)). \tag{2}$$

The Offsets



What Have We Learned?

1. "Edge effects" cannot be ignored, even in the $n \longrightarrow \infty$ limit.
2. However, inter-post region rapidly independent of causet size.
3. Standard deviation appear $\propto \sqrt{n}$; proof remains elusive.
4. New (?) estimate on Euler function:

$$\lambda_{n-1} - \lambda < q^n.$$

To mathematicians this is the most interesting part of paper.

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5. Suggestions and questions?